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# The coupling of zero-dimensional exciton and photon states: a quantum dot in a spherical microcavity

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**Abstract.** Exciton-light coupling in spherical microcavities containing a quantum dot has been treated by means of classical electrodynamics within the non-local dielectric response model. Typical anticrossing behavior of zero dimensional exciton–polariton modes has been obtained, as well as the weak coupling — strong coupling threshold.

## Introduction

Since the first report of the strong coupling in quantum microcavities by Weisbuch et al [ ], a huge number of papers devoted to exciton–polaritons in planar microcavities have appeared. Strong enhancement of the light-matter coupling strength in these structures has been demonstrated, both experimentally and theoretically. Recent progress [ ] in photonic crystal fabrication gives hope that the four-decade long progression to lower dimensionality in exciton–polariton systems will soon achieve its logical conclusion with the appearance of a photonic dot with an embedded electronic quantum dot (QD). In particular, technological advances in the fabrication of spherical objects by the techniques of colloidal chemistry [ ] will hopefully provide a means for the practical fabrication of multilayered structures of spherical symmetry in due course. A rigorous theoretical analysis of an ideal system that exhibits coupling of zero-dimensional photons and excitons seems timely. We consider a spherical QD embedded in a spherical microcavity (SMC), as shown in Fig. 1(a).

## 1. Basic equations

A spherical electromagnetic wave can be represented as a superposition of two waves with decoupled polarizations [ ]: a TE wave with components $H_r$, $E_\theta$, $E_\phi$, $H_\theta$, $H_\phi$, and a TM wave with components $E_r$, $E_\theta$, $E_\phi$, $H_\theta$, $H_\phi$. The spatial dependence of the electric and magnetic fields in a spherical wave can be expressed in terms of spherical harmonics characterized by a positive integer $l$, and an integer $m$, in the interval from $-l$ to $l$, which are related to the angular orbital momentum and its projection.

Only in the case of TM mode with $l = 1$, is the electric field of the eigenmode not equal to zero at the centre of the SMC. The electric field of TM mode with $l = 1$, $m = 0$ near the centre of the SMC is spatially uniform and is directed along the $z$-axis, as illustrated in Fig. 1(b). Hence, only with this mode is there significant interaction with a (nonmagnetic) quantum dot placed at the centre of the microcavity. For all other cavity modes the electric
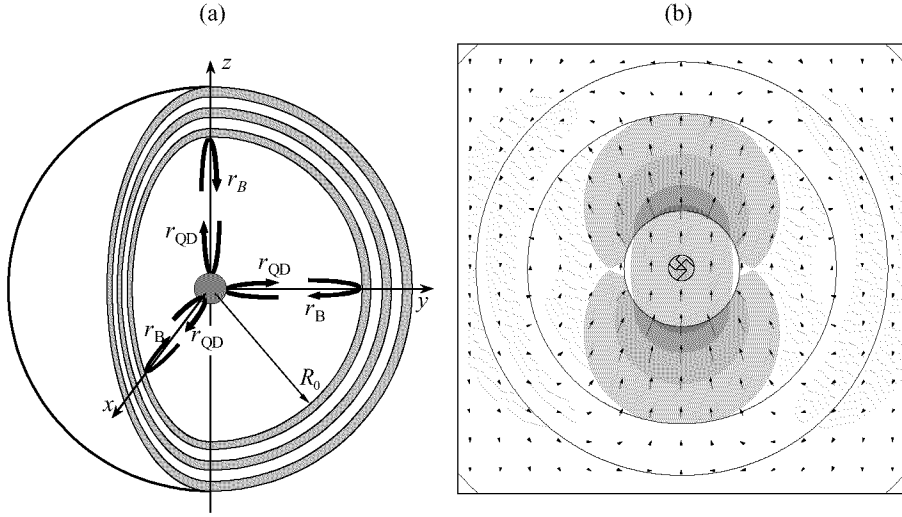
**Fig. 1.** (a) A schematic diagram of a spherical microcavity with a quantum dot at its centre. A central core of radius $R_0$ with the refractive index $n_0$ is surrounded by a spherical Bragg reflector, constructed from alternative layers of refractive indices $n_1$ and $n_2$. (b) The distribution of the electric field in a cross-section of the spherical microcavity for the TM mode with $l = 1, m = 0$.

field at the centre of the SMC vanishes, and there is negligible interaction with a QD placed there.

The electromagnetic field in the vicinity of a QD is described by Maxwell's equations with the excitonic contribution to the polarization $\vec{P}(\vec{r})$ taken into account:

$$\nabla \times \nabla \times \vec{E} - \varepsilon k_0^2 \vec{E} = 4\pi k_0^2 \vec{P}(\vec{r}). \tag{1}$$

where $k_0 = \omega/c$ and the polarization $\vec{P}(\vec{r})$ is coupled to the non-local excitonic susceptibility $\tilde{\chi}$ by $\vec{P}(\vec{r}) = \int \tilde{\chi}(\omega, \vec{r}, \vec{r}')\vec{E}(\vec{r}) \, d\vec{r}'$.

Further, using a Green function approach [ ] we can obtain an amplitude reflection coefficient for the TM wave with $l = 1$ incident to the quantum dot in the form

$$r_{QD} = 1 + \frac{2i\Gamma_0}{\omega_{ex} - \omega - i\,(\Gamma + \Gamma_0)}. \tag{2}$$

where the radiative damping factor $\Gamma_0$ is defined by the geometry of the exciton wave function, transverse-longitudinal splitting and the excitonic Bohr radius.

Connecting the electromagnetic field near the quantun dot and at the boundary of the central core by the transfer matrix method we can obtain an equation for the frequencies of the eigenmodes of the SMC with an embedded quantum dot:

$$h_1^{(2)}(kR_0) = r_{BR}r_{QD}h_1^{(1)}(kR_0). \tag{3}$$

where $h_1^{(1,2)}$ are the spherical Bessel function. When the central core radius exceed the wavelength of the light the spherical function and the phase of the reflection coefficient of the Bragg reflector can be approximated by their asymptotic values [ ] that allows us to rewrite equation (41) in the simplified form

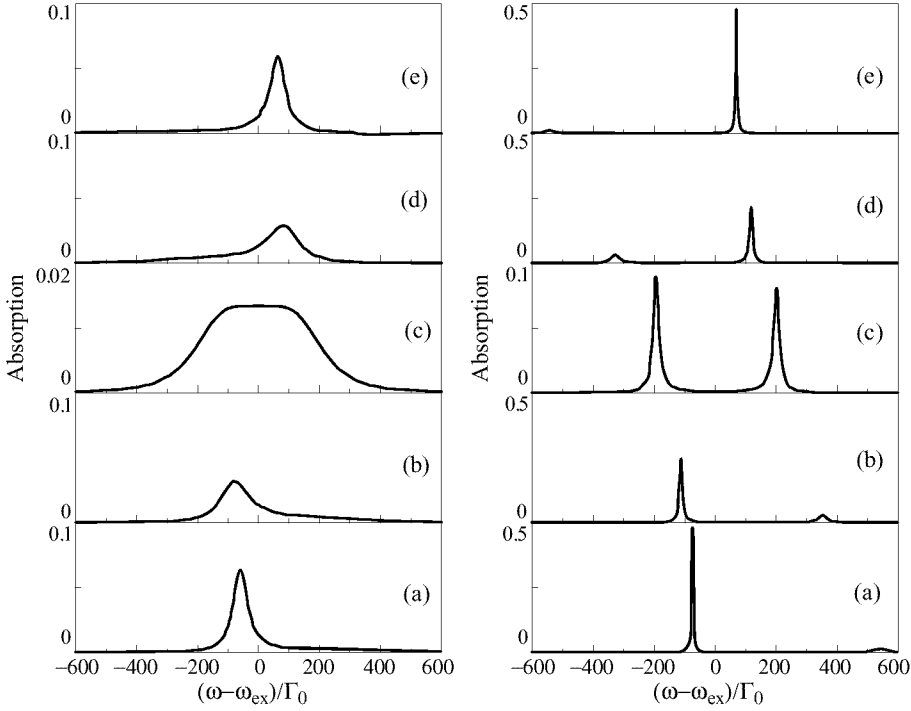$$(\omega - \omega_N)\,(\omega - \omega_{ex}) = (\Delta/2)^2 \tag{4}$$

**Fig. 2.** Absorption spectra of a spherical microcavity with a 5-period thick Bragg reflector (weak coupling regime, left picture) and 7-period thick Bragg reflector (strong coupling regime, right picture) The five spectra relate to different values of the central core radius $R_0$; (a) $R_0\omega_{ex}/2\pi c = 0.10265$; (b) $R_0\omega_{ex}/2\pi c = 0.1028$; (c) $R_0\omega_{ex}/2\pi c = 0.102955$; (d) $R_0\omega_{ex}/2\pi c = 0.1031$; (e) $R_0\omega_{ex}/2\pi c = 0.10325$.

where the value of the Rabi splitting is $\Delta = 2\sqrt{2\Gamma_0\omega_b/\left(b + 2\frac{R_0}{c}\sqrt{\varepsilon}\omega_b\right)}$ and $b = \pi n_1 n_2/\sqrt{\varepsilon}(n_2 - n_1)$.

## 2. Results and discussions

The interaction between localized exciton and photon modes has two different regimes; namely the strong coupling regime that holds when the splitting of the modes exceeds the half-sum of their damping parameters, and two peaks can be distinguished in the absorption spectrum, and the weak coupling regime that holds when the half-sum of the damping parameters exceed the splitting and the two peaks in the absorption spectra merge into one. In the case of a realistic quantum well or quantum wire exciton, the non-radiative damping of the exciton is usually much larger than the radiative term. In contrast, due to a discrete energy spectrum, the non-radiative damping of a quantum dot exciton is very small, and comparable with the radiative damping [ ]. Therefore the lifetime of the zero-dimensional polariton in this case is governed by the quality factor (Q-factor) of the spherical Bragg microcavity.

Figure 2 shows the absorption spectra, calculated using the transfer matrix method, of the two SMC with Bragg reflectors with thicknesses: 5-period (left picture) and 7 periods (right picture). The refractive index of the central core is 2.7 while the refractive indices

of the layers forming the Bragg reflector are 1.45 and 2.7 and correspond to the materials ZnTe and $SiO_2$, which are materials whose layers can be deposited by means of colloidal chemistry. The parameters of the QD are chosen to be similar to those for realistic QDs based on a II-VI semiconductor compound: $\Gamma_0 = 2 \times 10^{-6}\omega_{ex}$, $\Gamma = 10^{-6}\omega_{ex}$. For the the cavity with a 5 period Bragg relector the width of the cavity mode exceed the Rabi splitting $\Delta$ and the weak coupling occurs which manifests itself as the one peak in the absorption spectra (left). The shape of the peak in absorption becomes symmetric in the case of the precise tuning of the exciton and cavity modes. Increasing the thickness of the Bragg reflector we deacrease the width of the cavity mode and for the SMC with a 7-period Bragg reflector one can see the two separated peaks in the absorption spectra as it should be in the case of the strong coupling regime.

## 3. Conclusions

The interaction of zero-dimensional excitons and photons has been analyzed theoretically using the theory of non-local dielectric response and the transfer matrix method. Light absorption by a single quantum dot has been analyzed and it is shown that the resonant excitonic absorption of the $l = 1$ TM spherical wave incident on the quantum dot is total when the non-radiative and radiative damping factors of the exciton are equal. An equation for the eigenenergies and an expression for the value of the vacuum Rabi-splitting for the zero-dimensional polariton have also been obtained. Absorption spectra for a specific type of structure have been obtained and the transition between the strong and weak coupling regime has been illustrated.

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